

Day 1 Problem Set Solutions

Methods Camp | University of Texas at Austin

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Day 1 Problem Set

This document contains solutions to all exercises from the Day 1 problem set. To reinforce your understanding, please try your best to complete these exercises independently before referring to the solutions.

Exercise 1: Functions and Operations

1.1 Let $f(x) = 5x - 4$. Compute $f(2)$ and $f(0)$.

1.2 Let $f(x, y) = 3x + y^2$. Compute $f(2, 3)$.

1.3 Simplify: $\ln(50) - \ln(10)$. (Hint: use a log rule.)

1.4 Solve for x : $\ln(x) = 2$. (Hint: rewrite in exponential form.)

1.5 A class survey of 6 students records number of siblings: $\{1, 0, 2, 3, 1, 2\}$.

- a. Write the total number of siblings using summation notation and calculate it.
- b. Compute the mean $\bar{x}$.

Solutions

1.1 $f(2) = 5(2) - 4 = 6$; $f(0) = -4$

1.2 $f(2, 3) = 3(2) + 3^2 = 6 + 9 = 15$

1.3 $\ln(50) - \ln(10) = \ln(50/10) = \ln(5) \approx 1.609$

1.4 $\ln(x) = 2 \Rightarrow x = e^2 \approx 7.389$

1.5 (a) $\sum_{i=1}^6 x_i = 1 + 0 + 2 + 3 + 1 + 2 = 9$; (b) $\bar{x} = 9/6 = 1.5$

Exercise 2: Ratios, Proportions, Rates & Percentages

A town has 5,400 women and 4,600 men.

2.1 Compute the sex ratio (males per 100 females).

2.2 Compute the proportion of the town that is female.

2.3 Last year the town recorded 27 burglaries. Compute the burglary rate per 100,000 residents. (Total population = 10,000.)

2.4 The town's unemployment rate fell from 8% to 6%.

a. What is the change in percentage points?

b. What is the percent (relative) change?

2.5 A city's child poverty rate falls from 20% to 15%. An abstract reads: "Poverty drops 5%." Is this headline accurate? Why or why not?

Solutions

2.1 Sex ratio = $\frac{4,600}{5,400} \times 100 \approx 85.2$ males per 100 females.

2.2 Proportion female = $\frac{5,400}{10,000} = 0.54$ (54%).

2.3 Burglary rate = $\frac{27}{10,000} \times 100,000 = 270$ per 100,000 residents.

2.4 (a) Percentage-point change: $6 - 8 = -2$ p.p. (b) Percent change: $\frac{6-8}{8} \times 100 = -25\%$.

2.5 The headline is imprecise: the rate fell by 5 *percentage points*, not 5 *percent*. The true relative decline is $\frac{15-20}{20} \times 100 = -25\%$.

Exercise 3: Logarithms and Income

A researcher studies how income relates to years of education (X), using a model of log income:

$$\ln(Y) = 9.00 + 0.08X$$

where Y is annual income in dollars.

3.1 For a respondent with $X = 12$ years of education, compute $\ln(Y)$.

3.2 Convert $\ln(Y)$ into Y (dollars) by exponentiating. (Hint: use the inverse of $\ln$.)

3.3 Now consider a respondent with 13 years of education. Compute their income.

3.4 The exponentiated coefficient on X is $e^{0.08} \approx 1.083$. Interpret this number in one sentence.

Solutions

3.1 $\ln(Y) = 9.00 + 0.08(12) = 9.00 + 0.96 = 9.96$

3.2 $Y = e^{9.96} \approx \$21,163$

3.3 $\ln(Y) = 9.00 + 0.08(13) = 10.04$; $Y = e^{10.04} \approx \$22,926$

3.4 $e^{0.08} \approx 1.083$: each additional year of education is associated with income being multiplied by about 1.08 — roughly an 8% increase in income, on average.